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SOLVING ISSUES OF PREDICTION OF NATURAL THREE-DIMENSIONAL EQUATIONS USING FINITE DIFFERENCES METHOD

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ABSTRACT

This study aimed to solve the three-dimensional differential equations that treat natural phenomena as the spread of a substance or the movement of fluids.
The three-dimensional fluid motion equations were determined in differential equations and solved numerically by the method of the specified differences, and to find an approximate solution using the Tyler expansion to find the value of the derivative at each point in the field to predict this movement and the use of the BISC language in finding the results of solving these differential equations.

INTRODUCTION:

We still need to study and solve a lot of equations governing the movement and proliferation because of their contact with a large human life, as it represents a great challenge to human intellect, curiosity and scientific ambitions.

Where it was not possible until not long ago from the solution of those equations of three dimensions, so the development of efficient computer solution to this system of equations to provide speed, accuracy and space store solution for long steps.

It is helped by the development of means of calculation to give the results in writing a program to simulate the predictive equations Steps to resolve proliferation in general using of the three-Dimensional numerical methods.

The discussion dealt with determining the equations governing the movement of three-dimensional flow of water in estuaries, bays and shallow seas and resolve differences in a manner limited to finding a solution to predict the approximate speed in three dimensions depending on the location, depth, time waveform.

THE AIM:

It is an access to the differential equations that represent a pure physical movement of water velocity to keep the x, y, z in closed bodies of water and theoretical as much as possible.

In addition to identifying ways of solving equations using three-dimensional differences Ltd. and computer simulation in writing a program to solve prediction equations governing the movement of proliferation or move in different environments.

THE IMPORTANCE OF RESEARCH:

Treat the natural phenomena in the spread of material or transport or movement of certain fluid (including liquids and gases), which lasted less than expected, because of the difficulty to the point of impossibility to reach a researcher on the results of data processing of large and difficult ways to solve mathematically as it requires long steps and takes many time common with the requirements of the area of speed. shortening the time of the work, and helps the great development to the ability of modern computers in speed and accuracy of the results to solve the equations of motion of several natural phenomena.

for example:

The spread of germs and its impact on the area of deployment or reduction. Control the speed of water movement to avoid floods and control the movement of navigation. Stop the transmission of pollution in the water or air. [6]

PREVIOUS STUDIES

1) Elliptic Partial Differential Equations of second order have been studied using some numerical methods by Mithqal Ghalib Yousef Naji(1999) .

This type of differential equations has specific applications in physical and engineering models. In most applications, first- order and second-order formulas are used for the derivatives.

In this work higher order formulas such as: seven-points and nine-points formulas are used.

Using these formulas will transform the partial differential equation into finite difference equations.

To solve the resulting finite difference equations the following iterative methods have been used: Jacobi method,

Gauss-Seidel method, Successive Over-Relaxation method (SOR) and Multi grid method.

In this works, we found that multi grid methods are the most efficient among all other methods. The execution time for multi grid methods is of order three while the other methods is of order five .[5]

(2) Shin Aoi and Hiroyuki Fujiwara (1999)they have formulated a 3D finite-difference method (FDM) using discontinuous grids, which is a kind of multi grid method.

As long as uniform grids are used, the grid size is determined by the shortest wavelength to be calculated, and this constitutes a significant constraint on the introduction of low-velocity layers.

We use staggered grids that consist of, on one hand, grids with fine spacing near the surface where the wave velocity is low, and on the other hand, grids whose spacing is three times coarser in the deeper region. In each region, we calculated the wave field using a velocity-stress formulation of second-order accuracy and connected these two regions with linear interpolations.

The second-order finite-difference (FD) approximation was used for updating.

Since we did not use interpolations for updating, the time increments were the same in both regions.

The use of discontinuous grids adapted to the velocity structure resulted in a significant reduction of computational requirements, which is model dependent but typically one-fifth to one-tenth, without a marked loss of accuracy. [11]

3) Mathematical Model for Mod flow by Shin Aoi and Hiroyuki Fujiwara by Sayed-Farhad Mousavi(2003) :

The mathematical model is a representation of the physical state of the basin are mathematical relationships between hydraulic and hydro geological transactions in the basin and the hydraulic compressor.

Built form on the two well known equations, namely: the law of Darcy equation and keeping the subject and the collection of these equations gives the partial differential equation for the unstable flow of the following: [9]

$$\frac{\partial}{\partial x}(k_{xx}\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(k_{yy}\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(k_{zz}\frac{\partial h}{\partial z}) - w = S_s\frac{\partial h}{\partial t}$$

Where:

k permeability of water for pregnant water trends in the coordinate (x, y, z) (LT-1), where L stands for a single distance and T stands for one time.

h hydraulic compressor at time t (L).

W end is a nutrition or the clouds in unit volume in time t (T-1).

Ss storage specified for the rule permeability (L-1).

t time (T).

Equation (1) with boundary conditions and initial conditions for the mathematical description of the system of the movement of groundwater. The analytical solution of this equation is rarely possible, except in very simple cases, and this necessitated the need to use numerical methods.

The above equation is very complex and cannot be resolved on its own terms of public analysis, although there are some analytical solutions for special cases, so it relied on numerical methods to solve them.

Has been resolved partial differential equation by using one of the previous numerical methods are known Finite differences method. In this method replaces the continuous center representative in the equation (1) a specific set of points (a) separate spatial and temporal, gives the water balance in relation to each cell based on the following equation hold:

$$\sum Q = S_S \frac{\Delta h}{\Delta t} \Delta v$$

Where:

Qi rate flowed to and from the cell (L3T-1)

ΔV cell volume (L3)

Δh compressor change over time (Δ T) L

And runoff from the cell (i, j-1, k) to(i, j, k) gives the relationship:

$$q_{i,j-1/2k} = k_{i,j-k/2} \Delta c_i \Delta v_k \frac{h_{i,j-1,k} - h_{i,j,k}}{\Delta^r_{j-1/2}}$$

Permissible form of linear equations of constraint resulting together with the methods of iteration. Uses the frequency at each period time is to run the mathematical model which is imposed at the outset the values of initial compressor in each cell, and after the direct solution is to obtain values closer to the actual solution of the equations.

(4) Suyang Zhang and Weizhong Daia (2007)they have developed a finite difference method for studying thermal deformation in a 3D thin film exposed to ultra short pulsed lasers.

The method, based on the parabolic two-step heat transport equations, accounts for the coupling effect between lattice temperature and strain rate, as well as for the hot electron-blast effect in momentum transfer.

By replacing the displacement components in the dynamic equations of motion using the velocity components, developing a fourth-order compact method for evaluating stress derivatives in the dynamic equations of motion, and employing a staggered grid, we have developed a numerical method that allows us to avoid non-physical oscillations in the solution.

Numerical results show the displacement and stress alterations at the center along the z direction, and along x and y directions, which reveal that the central part of thin film expands.

Further research will focus on 3D double-layered cases where the interface could be either perfect thermal contact or imperfect thermal contact.[12]

EQUATIONS PROPOSED SOLUTION:

The mathematical formula for many of the issues on the boundary of applied ordinary differential equations, or partial differential equations, has expanded roads and numerous ways to find analytical solutions of these equations that are usually difficult or impossible, especially partial differential equations, Because the use of numerical method to find approximate solutions of these equations ,depends on more or less energy calculator storage and speed the implementation of the accounts ..

Determine the equations governing the movement of currents in shallow water in the following formulas: [2], [3]

$$f_t + f_x^2 + (fg)_y + (fh)_z - Ff + \rho^{-1} p_x - \rho^{-1}(\psi_{xxx} + \psi_{xyy} + \psi_{xzz}) = 0 \quad \ldots(1)$$

$$g_t + g_y^2 + (fg)_x + (gh)_z + Ff + \rho^{-1} p_y - \rho^{-1}(\psi_{yxx} + \psi_{yyy} + \psi_{yzz}) = 0 \quad \ldots(2)$$

$$p_z + \rho l = 0 \quad \ldots(3)$$

Where: -
x, y, z axis represent the coordinates
f, g, h the speed of the wave directions x, y, z respectively
t time (seconds)
F = Korieuls F acceleraion
density $= \rho$

Z

NUMERICAL METHODS TO SOLVE DIFFERENTIAL EQUATIONS:

Must convert the partial differential equation, that describes the physical phenomenon which, already mentioned to a specific mathematical model, can be applied to numerical methods appropriate for the approximate values around that model.

Known as numerical modeling as a convert mathematically (partial differential equations) to the image of an integer for ways to bring the differential equation and replace it, with an approximate value in the form of discrete domain of the elements or (nodes) network numerical representation (grid) and there are several ways to do this rounding of the most important finite difference method , and boundary element method where this approximation converts the differential equation with variable related to a set of algebraic equations which gives the dissolution values of the variable considered in the contract the network used. [4]

FINITE DIFFERENCES

The digital technology to find approximate solutions of partial differential equations are numerically using standard techniques such as Euler method, Runge Kutta, etc.. [1]
The solution of partial differential equations, the main challenge is to create a convergence of the equation to be studied, but numerically stable, and this means that the errors of inputs and intermediate calculations do not accumulate and cause the resulting output to be meaningless.
And is characterized by the following: -

Capability of modeling the complex issues that support the three dimensions.

That the accounts in this way is testing the appropriate elements in the form of a rectangle or a triangle or any other curved shape to cover all points of the region solution.

Simulation can be made more easily Engineering irregular border.

Also characterized by element method efficiency in determining the border areas of the form that represents the study.

As well as in the representation of variables affecting, for example, in the behavior of fluid such as density and viscosity.

The ability to calculate the variables in the points located within the elements and by updating the values of those variables that were calculated for a single element in the contract Nodes The weaknesses in the finite element method can be diagnosed as follows: -

Lack of style and clear of this method is dependent experience.

Generated in this way the number of matrices that must be solved using efficient numerical methods to solve systems of linear equations or non-linear.
Require complex calculator program that can increase the cost of use of the method compared with the way the differences Limited.

FINITE DIFFERENCE METHOD

The finite difference method (FDM) was first developed by A. Thom* in the 1920s under the title "the method of square" to solve nonlinear hydrodynamic equations.
The finite difference techniques are based upon the approximations of differential equations by finite difference equations. These finite difference approximations are algebraic in form, and the solutions are related to grid points. Thus, a finite difference solution basically involves three steps:
1. Dividing the solution into grids of nodes.
2. Approximating the given differential equation by finite difference equivalence that relates the solutions to grid points.
3. Solving the difference equations subject to the prescribed boundary conditions or initial conditions.[12]

As the most used method explicit way to solve partial differential equations. Easy preparation of the mathematical model and the relative simplicity the best ways to test the solution .Characterized by easily programming.

In this method the division of search space variables to small areas by using the parallel lines cross and the other observed in this study we used the tripartite division lines pivotal third intersect perpendicular to be the intersection of lines of these so-called contract NODES and all the variables that concern us to know their values are defined in the contract.

As for the values of those variables in the points located between the nodes can update their knowledge using mathematical Interpolation, an increase of the number of nodes and reduce the distance between two nodes.

we have used the expansion Taylor to find the value of the derivative for each point within the area and there are three versions of key for the derived partial first example in point n, m)) and depending on the loose series of Taylor (Taylor series expansion) as follows: - [10]

The central difference, backward differences and forward differences formulas respectively are

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$$f_x = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} \qquad ..(4)$$

$$f_x = \frac{f_{i,j} - f_{i-1,j}}{\Delta x} \qquad ...(5)$$

$$f_x = \frac{f_{i+1,j} - f_{i,j}}{\Delta x} \qquad ...(6)$$

It Can increase the accuracy of partial derivatives to increase the number of points used, but we have to forget that the increase in the number of points surrounding the point that we might be required to account for problems when you use the computer because of the increased storage capacity on the one hand and slow implementation on the other.

FORMULATION FINITE DIFFERENCES :

Direct compensation in the way explicit method to solve equations to move from first to third representation of velocities and pressure for each site to be determined on juvenile Network Ltd. following differences for the arbitrary variable F: -

$$F = F(i\Delta x,\ j\Delta y, k\Delta z, n\Delta t)$$

Which expresses the pressure P format differences front and on each of the accelerated current U,V,W in X,Y,Z Respectively, the central difference formula The change accelerated the current U, V for time t Format differences are the front and this What is common in dealing with the time of forecasting a new time is coming, depending on the current time.

We derived mathematical formulas for equations 1, 2 and 3 using the finite differences as follows:

Approximation will be to offset the momentum in the direction X as follows:

$$f_t + f_x^2 + (fg)_y + (fh)_z - Ff + \rho^{-1} p_x = 0$$

$$f_t = Ff - f_x^2 - (fg)_y - (fh)_z - \rho^{-1} p_x$$

$$\frac{f_{i,j,k,n+1}-f_{i,j,k,n}}{\Delta t}=Fg_{i,j,k,n}-\frac{f^2_{i+1,j,k,n}-f^2_{i-1,j,k,n}}{2\Delta x}-\frac{f_{i,j+1,k,n}g_{i,j+1,k,n}-f_{i,j-1,k,n}g_{i,j-1,k,n}}{2\Delta y}-$$
$$\frac{f_{i,j,k+1,n}h_{i,j,k+1,n}-f_{i,j,k-1,n}h_{i,j,k-1,n}}{2\Delta z}-\rho^{-1}\frac{P_{i+1,j,k,n}-P_{i-1,j,k,n}}{2\Delta x}$$

$$f_{i,j,k,n+1}=f_{i,j,k,n}+\Delta t*\left\{\begin{array}{l}Fg_{i,j,k,n}-\frac{f^2_{i+1,j,k,n}-f^2_{i-1,j,k,n}}{2\Delta x}-\frac{f_{i,j+1,k,n}g_{i,j+1,k,n}-f_{i,j-1,k,n}g_{i,j-1,k,n}}{2\Delta y}\\-\frac{f_{i,j,k+1,n}h_{i,j,k+1,n}-f_{i,j,k-1.n}h_{i,j,k-1.n}}{2\Delta z}-\rho^{-1}\frac{P_{i+1,j,k,n}-P_{i-1,j,k,n}}{2\Delta x}\end{array}\right\}$$

The same way as an approximation will be to offset the momentum in the direction Y

$$g_t+(fg)_x+g_y^2+(gh)_z+Ff+\rho^{-1}p_y=0$$

$$g_t=-Ff-(fg)_x-g_y^2-(gh)_z-\rho^{-1}p_y$$

$$g_{i,j,k,n+1}=g_{i,j,k,n}+\Delta t.\left\{\begin{array}{l}-Ff_{i,j,k,n}-\frac{f_{i+1,j,k,n}g_{i+1,j,k,n}-f_{i-1,j,k,n}g_{i-1,j,k,n}}{2\Delta x}-\frac{g^2_{i,j+1,k,n}-g^2_{i,j-1,k,n}}{2\Delta y}-\\\frac{g_{i,j,k=1,n}h_{i,j,k+1,n}-g_{i,j,k-1,n}h_{i,j,k-1,n}}{2\Delta z}-\rho^{-1}\frac{P_{i,j+1,k,n}-P_{i,j-1,k,n}}{2\Delta y}\end{array}\right\}$$

Is described by the pressure outside of the apportionment of the central simple relationship with the speed of the momentum equations in the first and second rounds and the pressure equation as follows: -

$$p_z+\rho l=0$$
$$p_{i,j,k+1,n+1}=p_{i,j,k-1,n-1}-2\Delta\rho l$$

SOLVE THE EQUATIONS BY USING BASIC PROGRAM:

The process of a Computer simulation Applications. Among the most important results is the representation of the various hydrological processes through the application and development of the mathematical model for the purpose of expression of the interaction time and then analyze and configure the hydrological cycle that has remained for a long time based experimental methods and statistical adopted the linking information in the relations of calculation far somewhat linked to hydrological processes that describe.

We have written words of the program language Basic to its potential in solving this system of equations and accept the three-dimensional and the same area Storage wide and repeating calculations, and speed of implementation and give the results of the Basic language is the most commonly used study for employees in areas far from the jurisdiction of the exact of the computer .

And added to clarify each of the phrases in the program as a message to the beneficiary for continuous work in solving this type of equations in general and to study the nature of estuaries exposed to constant changes, in particular [4].

Can trace the following steps to understand the program and noting the wide flexibility to enter or change any step of it: -

1 - Reservation dimensional i, j, k for each of the directions x, y, z respectively, the maximum size of memory storage borne calculator to represent the maximum length, width and depth.

2 - Open files to store raw results for later, which is accelerated schedules for f, g, h in terms of time and the name of each file is the names of the dimensions of the node you want to know where velocities.

3 - to give initial conditions for all other boundary points and velocities are closer to reality.

4 - adopted changes in the length of the channel first and then the width of the channel and thus the depth of the bottom to the surface in the calculation of velocities f_1, g_1, h_1 to the following time of each coordinate for i, j, k then alter the values of f, g, h values of velocities f_1, g_1, h_1 to respectively, to restore the accounts for the cycle time subsequent to the last cycle of time can give the factual findings by the possibility of accelerated computer Storage.

5 - Calculation of the outcome of the velocities by installing the three velocities f, g, h to get an approximate value.

A PROSPECTIVE STUDY:

The equations of Navier – Stokes is also important , because of the wide applications, where to this day has failed to demonstrate a lasting solution to the equations of Navier - Stokes in three-dimensional space .

Where is called this group of issues the name of the issues and the existence and flow of Navier - Stokes, one of the issues of the twenty-first century put forward by Clay Mathematics Institute and offered a prize.

And the Navier-Stokes equations of several versions, where there are several ways to express equal differential formulas. [10]

The following is the formula for this differential formulas:

• equation of communication (or equivalent output of the cluster):

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

output equation of the amount of movement :

$$\frac{\partial (\rho \vec{v})}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} \otimes \vec{v}) = -\vec{\nabla} p + \vec{\nabla} \cdot \overline{\overline{\tau}} + \rho \vec{f}$$

equation of energy output:

$$\frac{\partial (\rho e)}{\partial t} + \vec{\nabla} \cdot [\, (\rho e + p)\, \vec{v}\,] = \vec{\nabla} \cdot \left(\overline{\overline{\tau}} \cdot \vec{v} \right) + \rho \vec{f} \cdot \vec{v} - \vec{\nabla} \cdot \vec{q} + r$$

In these equations:

- **t represent the time**
- p **represent the volumetric mass of fluid**

- $\vec{v} = (\, v_1,\ v_2,\ v_3\,)$ **The speed of the molecule fluid .**
- p **The pressure**

CONCLUSIONS

1) The study showed that the solution of three equations which represent the movement of water flows by using the finite differences method are possible in spite of the many prefer to the use of one-dimension or two dimensions studies.

2) Using the finite differences are possible method to predict the three-dimensional movement of water flow in rivers that represent the three equations have been solved.

3) Using a computer to write the program by a Basic easy testing solution to the equations of the three that have been solved.

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